USVMC 2014–2015
Fun with arithmetic!

(1) Using exactly four 4s (and no other digits at all), the standard arithmetic operations, and whatever standard mathematical notation you like (such as fraction bars, square roots, factorials, decimal points, parentheses and exponents), show how to express all of the numbers from 2 to 10. Concatenation is also acceptable, meaning that, if you wish, you may take two of the 4s and turn them into the number 44.

(2) When I was a kid, my brother and I used to play a game called Krypto. Five cards were dealt onto the table, face up, each showing a number between 1 and 25. A sixth card was then dealt face-up, beside the first five. The goal was to use the standard arithmetic operations to manipulate the five given cards into the sixth. Standard mathematical notation (such as factorials, exponents, square roots, parentheses, and decimal points) was permitted, but concatenation was not allowed. For example, if the five given numbers were $1, 2, 3, 4, 5$ and the goal was to make 6, then here are two possibilities:

$$6 = 3(5-4) + 2 + 1$$
$$6 = (5+1)(2+3-4)$$

Solve each of the following Krypto hands:

(a) Given:1,2,5,6,17. Goal: 8.
(b) Given:1,4,9,15,20. Goal: 10.
(c) Given:2,3,5,6,21. Goal: 12.
(d) Given:5,7,11,19,23. Goal 1.
(e) Given:18,20,21,22,25. Goal 3.

Note that you are required to use all five numbers in your solution.

(3) Find a four-digit number whose digits reverse upon being multiplied by four.

(4) Find a three-digit number that is equal to the sum of the factorials of its digits.

(5) In the addition problem below, each letter stands for a different digit. However, each letter stands for the same digit in every place where it appears. Find the only possible value for each digit to make a correct addition statement:

$$\begin{array}{r} \text{S E N D} \\ + \quad \text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

(6) Using the digits from 1-9 exactly once each, what are the two numbers which multiply to give the largest possible product?

(7) Using the digits from 1-9 exactly once each, form three, three-digit numbers such that one of them is the sum of the other two.

(8) Find the largest number which, when written out in English, has its letters in alphabetical order.